Thermodynamics I

System of Units (SI)

- Force (derived)

$$F \propto m \cdot a$$

$$F = k \cdot m \cdot a$$

$$k = \underline{1}$$

$$1N = 1kg \cdot 1\,m/s^2 = 1\,kg\cdot m/s^2$$

- Pressure (derived)

$$P = \frac{\text{Normal Force}}{\text{Area}} = \frac{F_N}{A} = \frac{1N}{1m^2} = 1\,N/m^2 = 1\,Pa$$

- Specific Volume

$$\nu = \frac{\text{Volume}}{\text{mass}} = \frac{\forall}{m} = \frac{1\,m^3}{1\,kg} = 1\,m^3/kg$$

$$\rho = \frac{\text{mass}}{\text{Volume}} = \frac{m}{\forall} = 1\,kg/m^3 \quad [\text{density}]$$

- Temperature

T= °C or °K = °C + 273.15

Heat (Q) is a form of energy transfered due to temp. difference.

Thermodynamic Properties

- Thermodynamic Properties - a physical entity/aspect which can be used to define/specify the Thermodynamic state of a system/substance.

examples:

- Mass, m (kg)
- Volume, $\forall$ (m^3) $1m^3 = \underline{1000}$ Litres
- Specific Volume, $\nu = \forall/m$ (m^3/kg)
- Temperature, T (°C or °K)

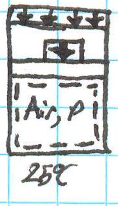
- Pressure, P(kPa)
- U, Internal Energy (1st Law of Thermo.)
- H, Enthalpy (1st Law of Thermo.)
- S, Entropy (2nd Law of Thermo.)

- In this course we deal strictly with pure substances in their liquid or gaseous state:
  - Pure Elements: O_2, N_2, ect.
  - Pure Compounds: H_2O, CO_2, NH_3, ect.

  except for air.

- Exactly two independant Thermodynamic Properties are required to uniquely define the Thermodynamic State of a pure substance. To know the Thermodynamic State is to know all Thermo. Properties of the substance.

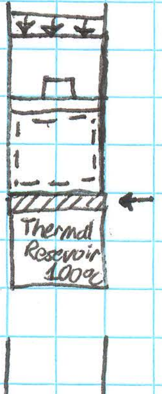
Overview of Thermodynamic Processes



- Equilibrium State

$$F_{up} = F_{down}$$
$$P_{atmo}A + W = P_{sys}A$$
$$P_{sys} = P_{atmo} + W/A$$

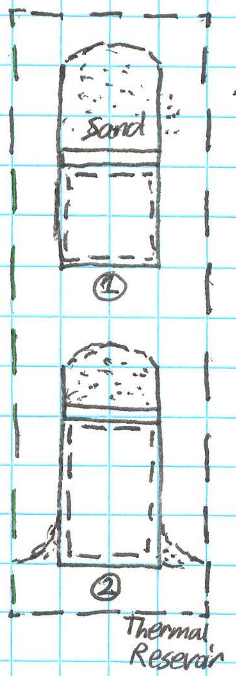
- Quasi - Equilibrium Process - a process which occurs slow enough such that a thermodynamic property does not change.



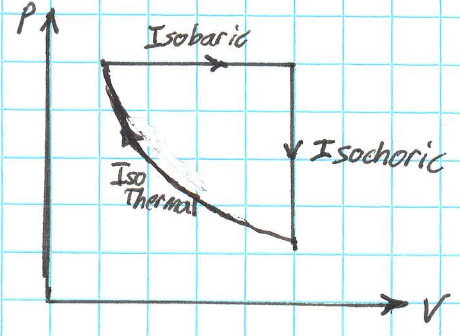
- Isobaric Process (P = constant)

insulation - Slows heat (transfer)

- Isochoric Process (V = constant)



- Isothermal Process (T=constant)

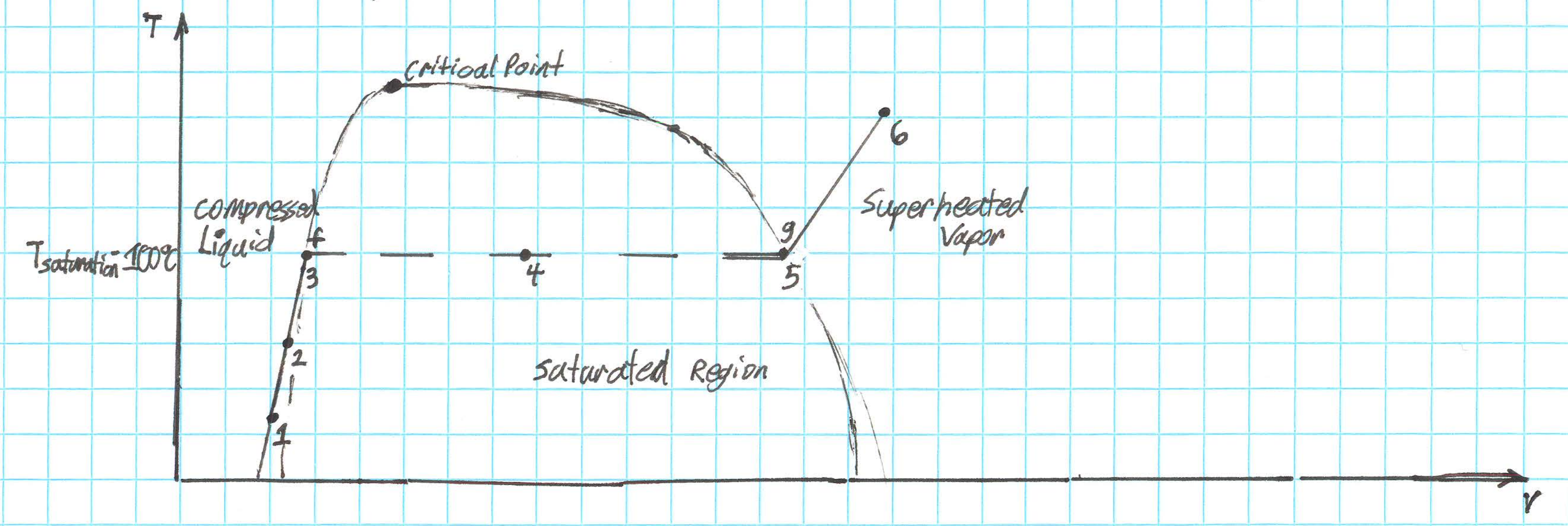


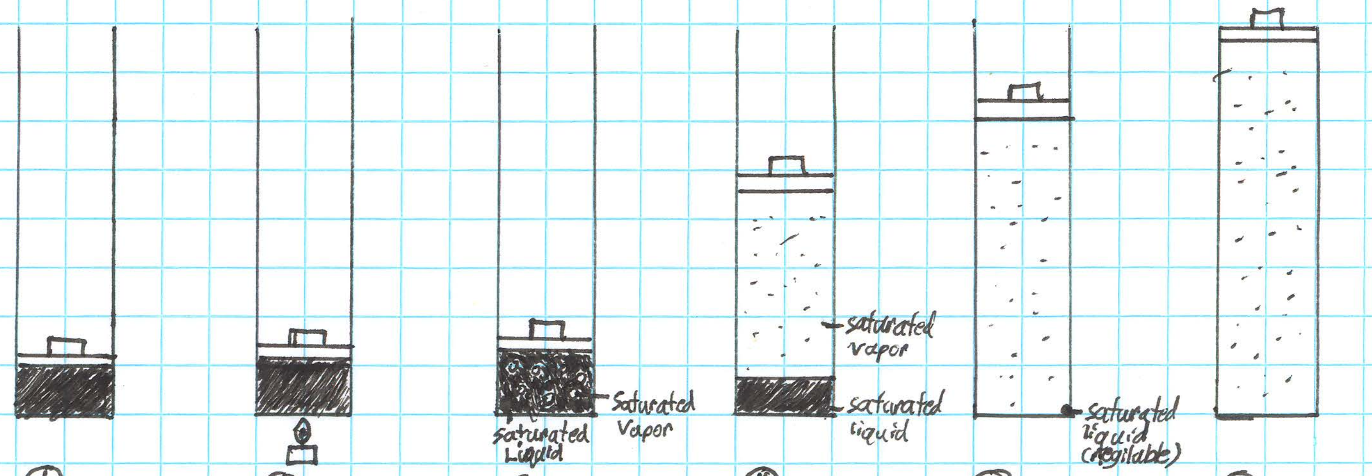
Note: DO NOT USE ideal gas law ($PV=nRT$) for the substances listed in Appendix B (Fundamentals of Thermo., Borgnakke, Sonntag) and R12/22. Everything not in tables can use ideal gas law for approximation.

Ammonia (NH_3)
R-410a - Highly Efficient Refridgerant
Methane (CH_4)

CR (Compression Ratio) = $\frac{\text{Maximum Volume}}{\text{Minimum Volume}}$

Example Problem: Illustration of Temp./Press. as dependant Thermo. Properties





①	②	③	④	⑤	⑥
pure liquid H_2O					
P_1	$P_2 = P_1$	$P_f = P_3 = P_2$	$P_4 = P_3 = P_f$	$P_5 = P_4 = P_f$	$P_6 = P_5$
T_1	$T_2 \gg T_1$	$T_f = T_3 \gg T_2$	$T_4 = T_3 = T_f$	$T_5 = T_f$	$T_6 \gg T_5$
$v_1 = V_1/m$	$v_2 > v_1$	$v_f = v_3 > v_2$	$v_g > v_4 \gg v_f$	$v_5 \gg v_4; v_5 = v_g$	$v_6 \gg v_5$
Compressed liquid		Saturated, $x=0$	$0 < x < 1$	$x = 1$	

ex. cont.

For saturated systems we define a Thermodynamic property, quality (x).

$$x = \frac{m_{gas}}{m_{total}} = \frac{m_g}{m_g + m_L}$$

$$V_{sys} = v_f(1-x) + v_g(x)$$

$P = 500\,kPa^*$ $\quad$ $* = \text{given}$

①	②	③
		$T = 151.86°C$
$T^* = 100°C$	$T^* = 140°C$	$V = .001093 = v_f$
$v = .001043$	$v = .00108$	$x \approx 0$
p. 790 [Compressed Liq.]		

④	⑤	⑥
$T = 151.86$	$T = 151.86$	$T^* = 300°C$
$v^* = .2$	$v = .37484 = v_g$	$V = .52256\ m^3/kg$
$x = .532$	$x \approx 1$	
p. 780 [Saturated H_2O Pres.]		p. 785 [Superheated H_2O]

sample calculations

$$V_{sys} = v_f(1-x) + v_g(x)$$
$$V_{sys} - v_f = (-v_f + v_g)x$$
$$v - v_f = (v_g - v_f)x$$
$$.2 - .001093 = (.37484 - .001093)x$$
$$x = .532$$

Ideal Gas Law (Various Forms)

Note: Only to be used for ideal gases (air) and substances not listed in tables in Appendix.

1. $P\forall = mRT$
2. $Pv = RT$
4. $P\forall = nRT$

$m = kg$
$T = {}^\circ K$
$P = kPa$
$\forall = m^3$
$R = \frac{kJ}{kg \cdot {}^\circ K}$

$v = \frac{\forall}{m}$

3. $P\bar{v} = \bar{R}T$

$\bar{v} = \frac{\forall}{n}$

$\bar{R}$ = Universal Gas Constant
n = kmole
$n = \frac{m}{M}$ M (molar weight)
$R = \frac{\bar{R}}{M}$

- For a fixed-mass Thermo. Process:

$$\frac{P_1 \forall_1}{T_1} = mR = \frac{P_2 \forall_2}{T_2} \quad \text{or} \quad \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

- When in actual practice is the Ideal Gas Law a good model?

Primary Rule:
- It must be a Gas. (duh?)

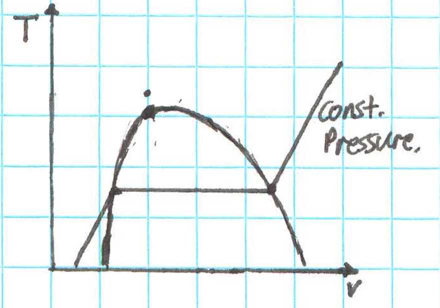
Secondary Rules:
- Low pressure - As pressure approaches zero all gases act as ideal gases.
- High temperature - $2x^+$ critical temperature if the pressure is not too high.

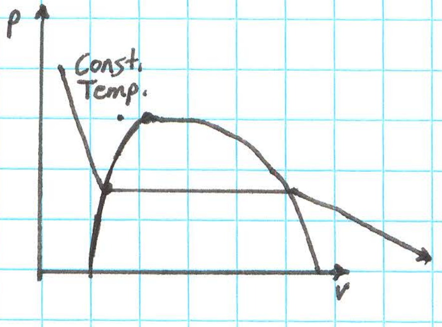
ex. air $P = 100 kPa$ (1 atm) $T = 25°C = 298°K$

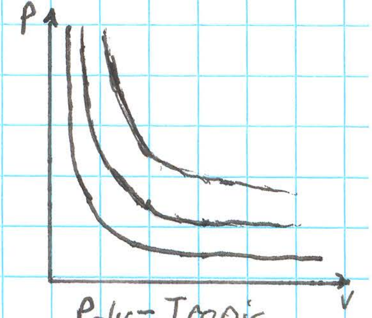
air ≈ 79% N_2 $T_{crit.} = 126.2°K$
$P_{crit} = 3390 kPa$

21% O_2 $T_{crit} = 154.6°K$
$P_{crit} = 5040 kPa$

Thus. High Temp./Low Pressure.





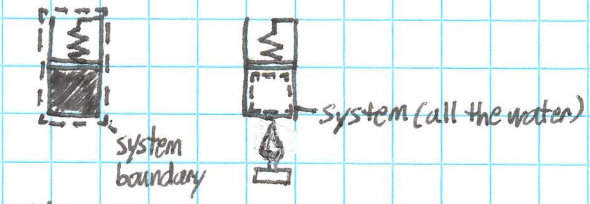


Poly-Tropic
Ideal Gas

Heat and Work:

- Heat (Q) (often redundantly refferred to as Heat Transfer) - Thermal energy transfer across a system boundary as consequence of a Temperature difference between the system and its surroundings.
- Work (W) - In classical Thermodynamics for fixed mass systems, all energy transfer across the system boundary which is NOT heat is classified as work. This is primarily composed of a force acting through a displacement. ($W = F \cdot L \Rightarrow 1\,\text{Joule} = N\text{-}m$)

ex. water as working fluid



No Work   Work

FBD of Piston:

$$W_p = \int_{①}^{②} F\,dl$$

$$= \int_{①}^{②} \frac{F}{A_{piston}}\,d\forall$$

$$= \int_{①}^{②} P_{ext.}\,d\forall$$

Similarities:

1. Both use the same units in the SI system (KJ).
2. Both are boundary phenomenona, which means both are modes energy enters/leaves the system.

3. Both are path functions. They depend on the initial and final states of the system and the details of the Thermo. process inbetween.

4. Being path functions, small quantites of each are represented by inexact differentials.

$$ {}_1Q_2 = \int_{①}^{②} \delta Q \neq Q_2 - Q_1 $$

$$ {}_1W_2 = \int_{①}^{②} \delta W \neq W_2 - W_1 $$

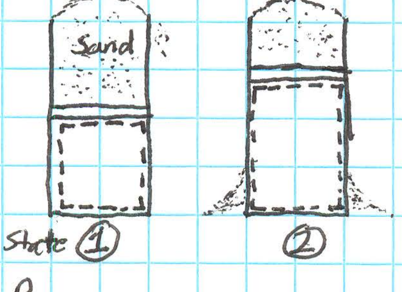
3/4 cont. Thermo. properties are point functions (vs. Heat/Work) and only depend on end points (initial and final states). Small changes in Thermo. Properties are represented by traditional differentials.

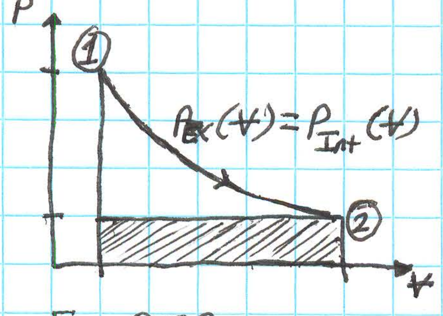
ex.

dT - Temp. $\int_{T_1}^{T_2} dT = T_2 - T_1$

- Quasi-Equilibrium (Process):

A Quasi-Equilibrium Process (QEP) is an ideal process which occurs slow enough that the system's deviation from thermodynamic equilibrium is infinitesimal.





The "sand" example explained:
In this example, one has a system of air, contained in a cylinder, topped with a piston and weighed down by sand. Now, sand is an infinitesimal small amount of mass. If one were to knock a couple grains of sand of the top of the pile, the piston would rise an infinitesimal amount, which would cause more sand to fall, ect. This continues at a slow rate, until the system is at equilibrium again.

For QEP:

$P_{External}(\forall) = P_{Intern.}(\forall)$

Thus

$$ {}_1W_2 = \int_{\forall_1}^{\forall_2} P_I(\forall)\, d\forall $$

- Sign Convention:
  Heat (Q):
    +Q = Heat into system
    −Q = Heat out of system
  Work (W):
    +W = Work out of system   Heat Engine
    −W = Work into system   Refridgerator/ Heat Pump/ A/C

- Polytropic Process (Quasi-Equilibrium Process):
  $P\forall^n = \text{constant}$
    n = polytropic exponent which characterizes the process

Since a polytropic process is a QEP

$$_1W_2 = \int_{\forall_1}^{\forall_2} P(\forall)\,d\forall$$

$$= \text{const.} \int_{\forall_1}^{\forall_2} \forall^{-n}\,d\forall$$

$n \neq 1$:

$$_1W_2 = \text{const.}\left[\frac{\forall_2^{-n} - \forall_1^{-n}}{1-n}\right]$$

$$\text{const.} = P_1\forall_1^n \ \underline{\text{or}} \ P_2\forall_2^n$$

$$_1W_2 = \frac{P_2\forall_2 - P_1\forall_1}{1-n}$$

$n = 1$ (Constant Temperature)

$$P_2\forall_2 = P_1\forall_1$$

$$\frac{P_2}{\forall_1} = \frac{P_1}{\forall_2}$$

$$_1W_2 = P_{(2/1)}\forall_{(2/1)}\ln\left(\frac{\forall_2}{\forall_1}\right) = -P_{(2/1)}\forall_{(2/1)}\ln\left(\frac{P_2}{P_1}\right)$$

$n = 0$ (Constant Pressure)

$n = \infty$ (Const. Volume): No Work

- First Law of Thermodynamics (for a Fixed-Mass System)
  - Thermodynamic Cycle: This is a cycle which starts and ends at the same Thermodynamic State; therefore is no net change in any Thermodynamic Property at the conclusion of the cycle.
  - First Law of Thermo. for a F-M (Fixed-Mass) system which undergoes a Thermo. Cycle is:

$$\oint \delta Q = \oint \delta W \quad \text{(S.I. Units Only)}$$

$$Q_{Net} = W_{Net}$$

This law is based soley on empirical (experimental) data and thus can not be derived, but from experimental data.

From the First Law of Thermo. for a F-M system which undergoes a Thermo. Cycle, it can be proven for an individual arbitrary Thermo. Process, the First Law of Thermo. for a F-M system is given by:

$$_1Q_2 = E_2 - E_1 + {_1W_2}$$

E = Total Energy of the System (kJ)

$= U + KE + PE$

$= U + \frac{1}{2} m V^2 + mgz$

U = Internal Energy (of molecules)

$$_1U_2 = m(u_2 - u_1)$$

u = specific internal energy (found in tables)

$$[{_1Q_2} + [U_1 + \tfrac{1}{2} m V_1^2 + mgz_1] = [U_2 + \tfrac{1}{2} m V_2^2 + mgz_2] + {_1W_2}$$

- Internal Energy (u) for Saturated Systems:

$$u = (1-x)\,u_f + (x)\,u_g$$

$$u = u_f + x\,(u_{fg})$$

- For a F-M system, if changes in kinetic energy (ΔKE) and potential energy are neglected, the First Law of Thermo. reduces to:

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

Enthalpy? There's a table for that

- Enthalpy (H):

$$H \equiv U + P\forall$$

Specific Enthalpy:

$$h \equiv \frac{H}{m} \equiv u + Pv$$

- Specific Heat:

Constant Volume Specific Heat:

$$C_\forall = \left(\frac{\partial u}{\partial T}\right)_\forall \qquad \partial u = C_\forall \cdot \partial T$$

Constant Pressure Specific Heat:

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad \partial h = C_p \cdot \partial T$$

- Specific Heat (For Ideal Gases):

$Pv = RT$ (Ideal Gas Law)

$h \equiv u + Pv$

$h = u + RT$

* $(h_2 - h_1) = (u_2 - u_1) + R(T_2 - T_1)$

* $C_{p_0}(T) = C_{v_0}(T) + R$

$h_2 - h_1 = \int_{T_1}^{T_2} C_{p_0} \cdot m \; dT$ (Table A.6)

$u_2 - u_1 = u_{T_2} - u_{T_1}$ (Air) (Table A7.1)

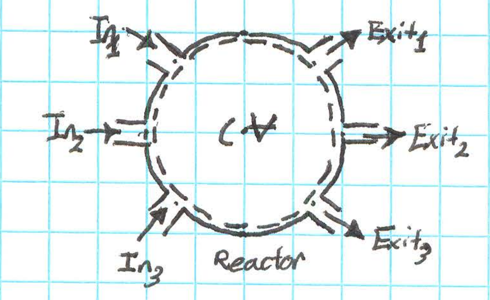
- Constant Specific Heat approximation:

$u_2 - u_1 = C_{v_0}(T_2 - T_1)$

$h_2 - h_1 = C_{p_0}(T_2 - T_1)$

Note: The smaller the Temp. variance and the higher the temp., the better this approximation is. Also this approx. is much more acurate for Nobel gases. (Fig. 5.11)

- First Law of Thermodynamics (for Control-Volumes)



Conservation of Mass:

$$\sum_{in} \dot{m}_i - \sum_{exit} \dot{m}_e = \frac{dm}{dt} = \dot{m}$$

$$(\dot{m}_{i_1} + \dot{m}_{i_2} + \dot{m}_{i_3}) - (\dot{m}_{e_1} + \dot{m}_{e_2} + \dot{m}_{e_3})$$

- First Law of Thermo for $C\text{-}\forall$:

$$\dot{Q}_{c\forall} + \sum_i \dot{m}_i (h_i + V_i^2/2000 + gz_i/1000) = \frac{dE_{c\forall}}{dt}$$

$$+ \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2000} + \frac{gz_e}{1000}\right) + \dot{W}_{c\forall}$$

where:

$\dot{Q}_{c\forall}$ - Heat Transfer Rate (kJ/sec = kWatts)

$\dot{W}_{c\forall}$ - Power (Rate of "Doing" Work) (kJ/sec = kWatts)

$\dot{m}$ - Mass Flowrate (kg/sec)

$$\dot{m} = \frac{V \cdot A}{v} = \frac{(m/s)\cdot(m^2)}{(m^3/kg)}$$

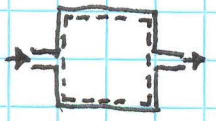
Two special cases for Control Volumes:

1. Steady-State, Steady Flow (SSSF)

$$\frac{d\text{"}x\text{"}}{dt} = 0, \quad x = \text{any property}$$

2. Uniform-State, Uniform Flow (USUF)
(Transient Problem)

1.b. Special Case of SSSF: Single Inflow and Outflow



Conservation of Mass becomes:

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

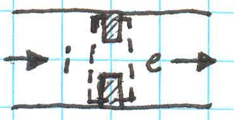
Conservation of Energy becomes:

$$\dot{Q}_{cv} + \dot{m}\left(h_i + \frac{V_i^2}{2000} + \frac{gZ_i}{1000}\right) = \dot{m}\left(h_e + \frac{V_e^2}{2000} + \frac{gZ_e}{1000}\right) + \dot{W}_{cv}$$

If ΔKE and ΔPE are neglected:

$$q_{cv} + h_i = h_e + w_{cv}$$

★ Throttling Valve: A throttling valve is designed to cause a pressure drop and is used in systems such as Air Conditioning (A/C)



SSSF case with single inflow and single outflow

Conservation of Mass:

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

Conservation of Energy:

$$\cancelto{0}{q_{cv}} + \left(h_i + \frac{V_i^2}{2000}\right) = \left(h_e + \frac{V_e^2}{2000}\right) + \cancelto{0}{w_{cv}}$$

There is no heat nor work leaving sys.
ΔPE neglected as $z_i = z_e$

ex. H_2O

$P_i = 2\,MPa$ $\quad$ $P_e = 100\,kPa$
$T_i = 500°C$ $\quad$ $T_e =$ $\quad$ or $\quad x_e$
$V_i = 10\,m/s$ $\quad$ $V_e = 20\,m/s$
$h_i = 3467.55\,\frac{kJ}{kg}$

$$\left(h_i + \frac{V_i^2}{2000}\right) = \left(h_e + \frac{V_e^2}{2000}\right)$$

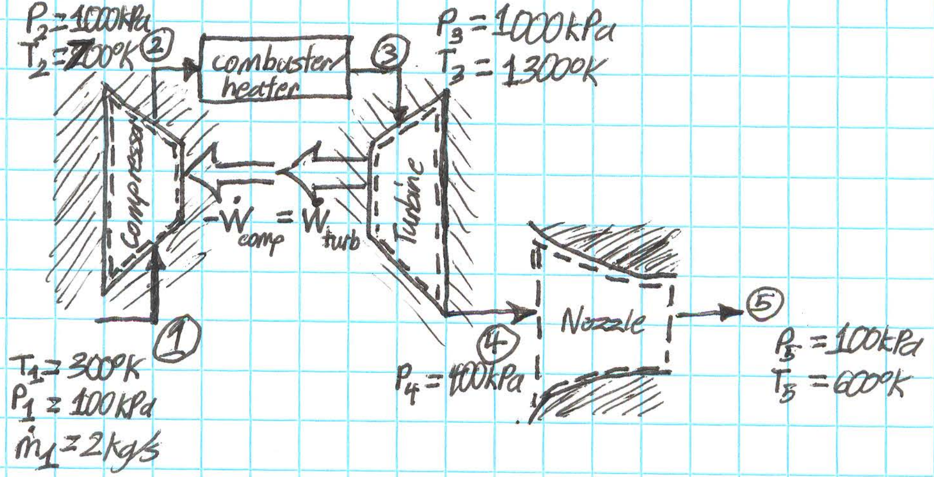
$$3467.55 + \frac{(10)^2}{2000} = h_e + \frac{(20)^2}{2000}$$

$h_e = 3467.4 \approx h_i$ [for this course $h_e = h_i$ for Throttling Valve.]

$h_e\ (@500°C) = 3488.09$
$\quad\ (@400°C) = 3278.11$

$T_e = 490°C$

ex. Jet Engine (Air is Working fluid)



Compressor, Turbine, and Nozzle are adiabatic

1-2: (Compressor)

$$\underset{\text{adiabatic}}{\cancelto{0}{q_{cv}}} + \left(h_1 + \cancelto{0}{\frac{V_1^2}{2000}} + \cancelto{0}{\frac{gz_1}{1000}}\right) = \left(h_2 + \cancelto{0}{\frac{V_2^2}{2000}} + \cancelto{0}{\frac{gz_2}{1000}}\right) + w_{cv}$$

$$h_1 = h_2 + w_{cv_{compresser}}$$

$$\dot{W}_{comp} = \dot{m}_1 (h_1 - h_2)$$

3-4: (Turbine)

$$\cancelto{0}{q_{cv}} + \left(h_3 + \cancelto{0}{\frac{V_3^2}{2000}} + \cancelto{0}{\frac{gz_3}{1000}}\right) = \left(h_4 + \cancelto{0}{\frac{V_4^2}{2000}} + \cancelto{0}{\frac{gz_4}{1000}}\right) + w_{cv_{turbine}}$$

$$w_{cv_{turbine}} = h_3 - h_4$$

$$\dot{W}_{cv_{turbine}} = \dot{m}_3 (h_3 - h_4)$$

$$-\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_3 - h_4)$$

$$\dot{m}_1 = \dot{m}_3 = \dot{m}$$

$$(h_2 - h_1) = (h_3 - h_4)$$

working fluid: Air (Ideal Gas)

$$\cancel{c_{p0}}(T_2 - T_1) = \cancel{c_{p0}}(T_3 - T_4)$$

$$T_3 - T_4 = T_2 - T_1$$

$$1300°K - T_4 = 700°K - 300°K$$

$$T_4 = 900°K$$

4-5: (Nozzle)

$$\cancelto{0}{q_{in}} + \left(h_4 + \cancelto{0}{\frac{V_4^2}{2000}} + \cancelto{0}{\frac{gZ_4}{1000}}\right) = \left(h_5 + \frac{V_5^2}{2000} + \cancelto{0}{\frac{gZ_5}{1000}}\right) + \cancelto{0}{W_{out}}$$

$$\Downarrow$$

$$|V_4 <<< V_5|$$

$$h_4 = h_5 + \frac{V_5^2}{2000}$$

$$V_5 = \sqrt{2000(h_4 - h_5)}$$

$$h_4 - h_5 = C_{P0}(T_4 - T_5)$$

$$= C_{P0}(300°K)$$

$$= 1.004(300°K) = 301.2$$

$$V_5 = 776.144 \, m/s$$

$$\text{Thrust} = \dot{m}(V_5 - V_1)$$

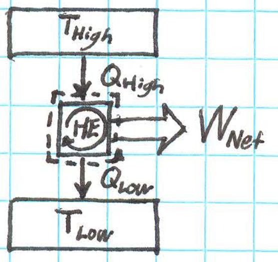
$$= 2 \, kg/s \, (776.144 \, m/s - 0)$$

$$= 1552.29 \, N$$

- Second Law of Thermodynamics (for Thermo. Cycles)
  - First Law of Thermodynamics for Thermo. Cycles:

    $$Q_{net} = W_{net}$$

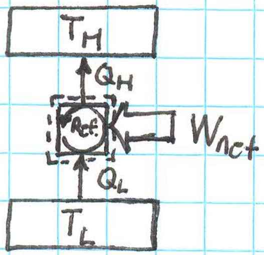
  - Thermal Reservoir (hypothetical): A hypothetical body which has an infinite "heat" capacity. Thermal energy in the form of heat can be input or removed from the resevoir in large amounts without changing the temperature of the resevoir.
    When the Thermal Resevoir produces heat, it is a Thermal Source.
    When the Thermal Resevoir absorbs heat, it is a Thermal Sink.
  - "Heat Engine": A Heat Engine (HE) is a system which operates in a Thermo. Cycle (1), Takes in heat from a higher temperature source (T_{High} / Q_{High}) (2), converts a fraction of Q_H into net work output (W_{net})(3), rejects the balance of Q_H not converted into work as waste heat, Q_{Low}, to a lower temperature (T_{Low}) thermal sink (4).



$$\oplus Q_{net} = \oplus W_{net}$$

$$\oplus Q_H + \ominus Q_L = \oplus W_{net}$$

  - Refridgerator / Air Conditioner (A-C) / Heat Pump:
    A system which:
    1. Operates on a Thermo. Cycle
    2. Takes in Thermal energy (Q_L) from a lower temperature (T_L) thermal source and moves it to a higher temperature thermal sink (T_H).
    3. Requires Work_{net} input to perform operation.



$$\ominus Q_{net} = \ominus W_{net}$$

$$\ominus Q_H + \oplus Q_L = \ominus W_{net}$$

- Kelvin-Planck Statement of the Second Law of Thermo.:
  (in application to Heat Energy)

  It is impossible to build a heat engine which:

  1. Rejects no heat (Q_L) to a Lower Temperature sink.
  2. Converts all Q_H from T_H into W_{net}
  3. Exchanges thermal energy with a single thermal reservoir
  4. Having Thermal Efficiency (η_{th}) = 1 (100%)

  Note: W_{net} can be $= 0$, thus $Q_H = Q_L$

- Clausius Statement of the Second Law of Thermodynamics:
  (in application to Refridgerators/Air Conditioning/Heat Pumps).

  It is impossible to build a Refridgerator or Heat Pump whose sole effect is the transfer of heat from a Low Temp. Reservoir (T_L) to a High Temp. Reservoir (T_H) without the input of work (W_{net}) nor is it possible for said refridge. or HP to have a Coefficient of Performance (β) = ∞

- Thermal Efficiency of a Thermo. Cycle:

$$\frac{E_{output}}{E_{input}}$$

For Heat Engines:

$$\eta_{th} = \frac{\oplus W_{net}}{\oplus Q_{High}}$$

For a Refridgerator/Air Conditioner:
(Coefficient of Performance)

$$\beta = \frac{\oplus Q_L}{\left| \ominus W_{net} \right|}$$

For a Heat Pump:

$$\beta' = \frac{\ominus Q_H}{\ominus W_{net}}$$

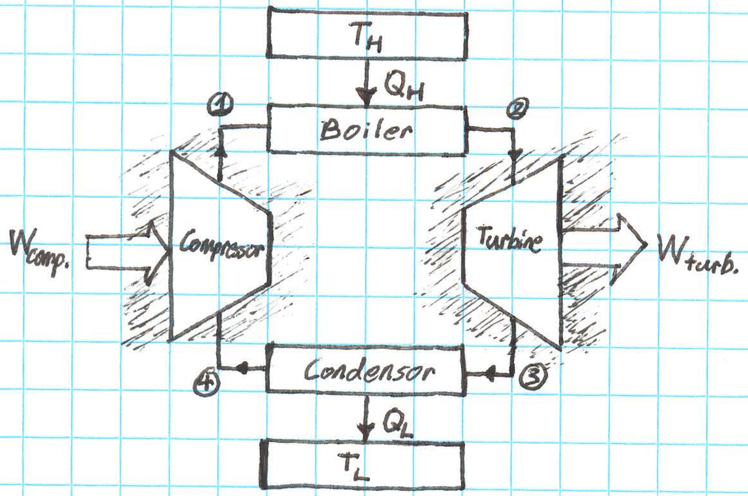
- Reversible Process: A Reversible Process is a process which occurs inmeasurably slow, such that at all times during the process, departures from Thermo. Equilibrium are infinitesimal. Thus this process is in Quasi-Equilibrium. Therefore at anytime during a reversible process, the process could be reversed, leaving the system and suroundings unchanged. (Theroretical)

$$W = \int P \, d\forall$$

- Irreversible Process: A process in which it is impossible to bring system and surroundings back to their original conditions. This is how all real-life processes work. Factors which cause irreversibility in real-life processes are:
  - Friction (Solid / Fluid)
  - Sudden Expansion / Compression
  - Heat transfer through a finite temperature difference.
  - Mixing of substances
  - Electrical Resistance
  - Fast Chemical Reactions

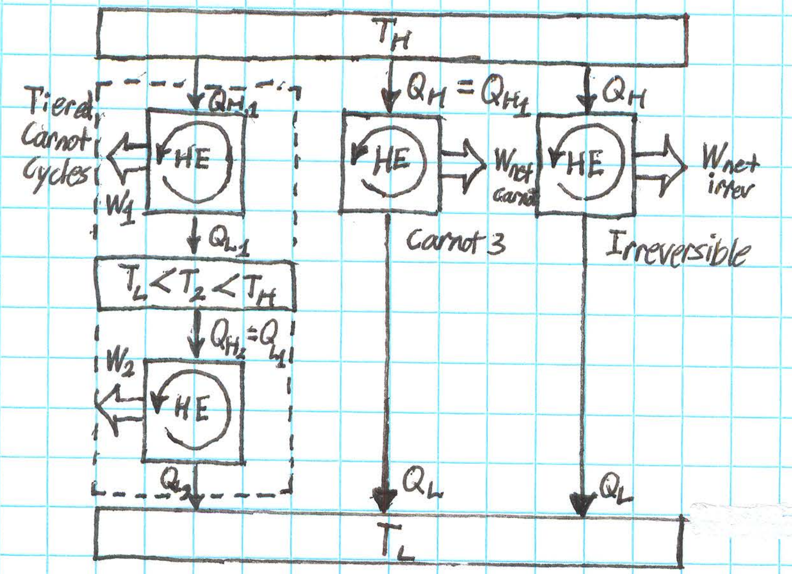
- Carnot Cycle : A Carnot Cycle is a special reversible Thermo. Cycle. It is defined by the following four reversible processes in order:

1-2. Reversible Isothermal process; Q_H transfered to the Working Fluid (WF) at T_H
2-3. Reversible Adiabatic expansion; WF goes from $T_H \rightarrow T_L$
3-4. Reversible Isothermal process; Q_L transfered from the WF at T_L
4-1. Reversible Adiabatic compresion; WF goes from $T_L \rightarrow T_H$



For a Carnot Cycle operating between two Thermal Resevoirs of given fixed T_H / T_L:

1. The thermal efficiency of the Carnot Cycle can not be exceeded by any cycle, reversible or irreversible.
2. The thermal efficiency of all Carnot Cycles, acting between a fixed and given T_H / T_L are the same and only depend on T_H / T_L. This includes tiered carnot cycles.



$$(\eta_{th})_{carnot\,3} = (\eta_{th})_{carnot\,1+2}$$

$$(\eta_{th})_{carnot} \geq (\eta_{th})_{\substack{irrev \\ reverse}}$$

It can be shown for Carnot Cycles (only):

$$\frac{T_L}{T_H} = \left|\frac{Q_L}{Q_H}\right| \quad \text{where } T_L/T_H \text{ are in } {}^\circ K$$

For any heat engine (including Carnot):

$$\eta_{th} = \frac{\oplus W_{net}}{\oplus Q_H} = \underset{\text{1st Law of Thermo}}{\frac{\oplus Q_{net}}{\oplus Q_H}} = \frac{\oplus Q_H + \ominus Q_L}{\oplus Q_H} = 1 - \left|\frac{Q_L}{Q_H}\right|$$

$$= 1 + \frac{\ominus Q_L}{\oplus Q_H}$$

Thus for Carnot Cycles (only):

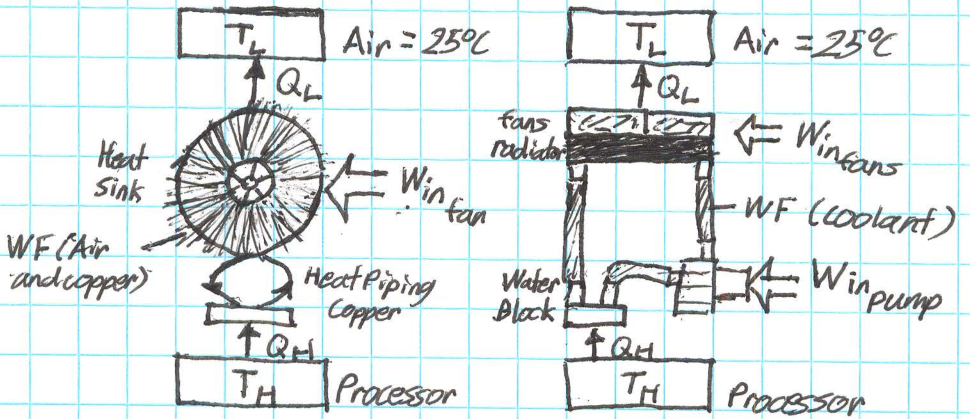
$$\eta_{th_{carnot}} = 1 - \frac{T_L}{T_H}$$

$$\underset{\text{(Refridgerator/ A/C)}}{\beta_{th_{carnot}}} = \frac{Q_L}{|W_{net}|} = \frac{Q_L}{|Q_{net}|} = \frac{Q_L}{|Q_H + Q_L|} = \frac{1}{\left|\frac{Q_H}{Q_L}\right| - 1}$$

$$= \frac{T_L}{T_H - T_L} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$\underset{\text{(Heat Pump)}}{\beta_{th_{carnot}}} = \frac{1}{1 - \left|\frac{Q_H}{Q_L}\right|} = \frac{T_H}{T_H - T_L} = \frac{1}{1 - \frac{T_L}{T_H}}$$

ex. Air vs Liquid Cooling



This is a real world example of air and liquid cooling systems in a tower computer. Prior to installing a liquid system, my GPU (Graphics Processing Unit) was running at 100°C on an air cooling system. After installing a liquid system the GPU hits a max of 70°C. Calculate and compare the coefficient of performance of each system. (Treat systems as if they were carnot).

$$\beta_{carnot_{air}} = \frac{(273.15 + 25°C)}{(273.15+100) - (273.15+25)} = \frac{T_L}{T_H - T_L}$$

$$= 3.975$$

$$\beta_{carnot_{LiqSys}} = \frac{T_L}{T_H - T_L} = \frac{298.15}{343.15 - 298.15} = 6.625$$

Nearly twice as efficient

- Second Law of Thermodynamics for a Fixed Mass-Sys. which undergoes Net state change

  - Inequality of Clausius: It can be proven that for any Thermodynamic cycle (Reversible or Irreversible):

$$\oint \frac{\delta Q}{T} \leq 0 \qquad \begin{matrix} < & \text{irreversible cycle} \\ = & \text{reversible cycle} \end{matrix}$$

Thus:

For a fixed mass system which undergoes a general Thermo. Process, having a net state change

$$\frac{\delta Q}{T} \leq dS$$

Where:

$S \equiv$ Entropy, a Thermodynamic property which is Path independant $(\mathrm{kJ}/{}^\circ\mathrm{K})$

$s \equiv$ Specific Entropy $(\mathrm{kJ}/({}^\circ\mathrm{K}\cdot\mathrm{kg}))$
$\equiv S/m$

Entropy? There's a table for that!

Thus:

$$\int_{①}^{②} \frac{\delta Q}{T} \leq \int_{①}^{②} dS$$

$$\leq S_2 - S_1$$

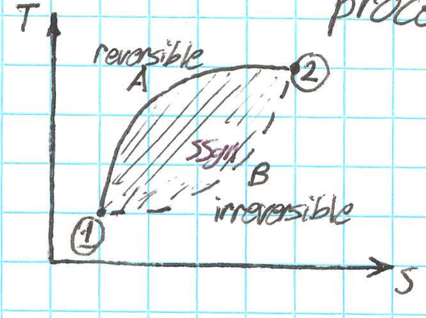
Yet, what is entropy?

Entropy is the most difficult Thermo. Property to define. Generally, entropy is defined as the chaos and expansion of a system and/or the universe.

$dS = 0$, for reversible adiabatic process

Thus:

$S_2 = S_1$, which is defined as an Isentropic process



$$\begin{array}{ccc} (S_2 - S_1)_A & = & \int_{\textcircled{1}}^{\textcircled{2}} \left(\frac{\delta Q}{T}\right)_A \\ \| & & \vee \\ (S_2 - S_1)_B & > & \int_{\textcircled{1}}^{\textcircled{2}} \left(\frac{\delta Q}{T}\right)_B \\ \text{Path Independant} & & \text{Path Dependant} \end{array}$$

Therefore:

We can deduce from the above, that for a fixed-mass system, there are only two physical phenomena which can cause the entropy of the system to change:

1. Heat Transfer (raise or lower entropy)
2. Irreversibility (raise entropy only)

Thus the Second Law can be rewriten as:
(Modern Form)

$$dS = \frac{\delta Q}{T} + \delta S_{gn}$$

where:

δS_{gn} = Entropy increase due to irreversibility, Always (+) and constant for a sys.

= 0 for Reversible Systems

$$S_2 - S_1 = \int_{\textcircled{1}}^{\textcircled{2}} \frac{\delta Q}{T} + {}_1 S_{gn\,2}$$

For a QEP (Reversible)

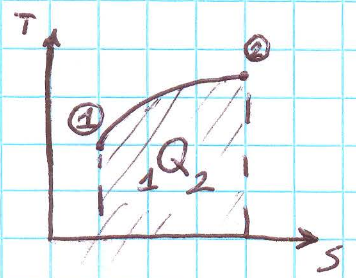
$${}_1W_2 = \int_{V_1}^{V_2} P(V)\, dV$$

In addition for Reversible processes

$$dS = \frac{\delta Q}{T}$$

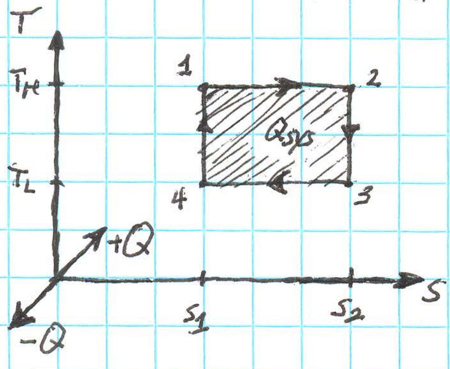
$$\delta Q = T\, dS$$

$${}_1Q_2 = \int_{S_1}^{S_2} T(S)\, dS$$



Recall Carnot Cycle:

1-2 Reversible Isothermal Process; Q_H at T_H
2-3 Reversible Adiabatic Process; $T_H \rightarrow T_L$
3-4 Reversible Isothermal Process; Q_L at T_L
4-1 Reversible Adiabatic Process; $T_L \rightarrow T_H$



$$Q_{sys_{net}} = Q_H - Q_L = W_{net}$$

↻(+) Heat Engine ↺(+) A/C, Heat Pump

For any reversible cycle, the area enclosed by it's cycle (on a T vs S diagram), represents W_{net} and Q_{net}.

- T-ds Equations

1st Law (for Fixed-Mass Sys.):

$$\delta Q = dU + \delta W \Rightarrow {}_1Q_2 = U_2 - U_1 + {}_1W_2$$

2nd Law (for Fixed-Mass Sys.)(Reversible Process):

$$\delta Q = T dS$$

$$\delta W = P d\forall$$

Combining the two laws gives us (valid if reversible or not):

$$T dS = dU + P d\forall$$

$$T ds = du + P dv \quad \text{(per unit mass)}$$

This is the first "T-ds" equation.

2nd "T-ds" Equation:

$$U = H - P\forall$$

$$dU = dH - P d\forall - \forall dP$$

$$dU + P d\forall = dH - \forall dP$$

$$T dS = dH - \forall dP \quad \text{(utilizing first "T-ds" eq.)}$$

$$T ds = dh - v dP$$

Change of Entropy for ONLY Ideal Gases:

$$Tds = du + Pdv$$

$$ds = \frac{du}{T} + \frac{P}{T}dv$$

$$du = C_{v_0}dT$$

$$ds = \frac{C_{v_0}dT}{T} + \frac{P}{T}dv$$

$$\frac{R}{v} = \frac{P}{T}$$

$$ds = \frac{C_{v_0}dT}{T} + \frac{R}{v}dv$$

$$s_2 - s_1 = \int_{T_1}^{T_2} C_{v_0}\frac{dT}{T} + R\ln\left(\frac{v_2}{v_1}\right)$$

$$Tds = dh - vdP$$

$$ds = \frac{dh}{T} - \frac{v}{T}dP$$

$$dh = C_{p_0}dT$$

$$ds = \frac{C_{p_0}dT}{T} - + \frac{v}{T}dP$$

$$\frac{R}{P} = \frac{v}{T}$$

$$ds = \frac{C_{p_0}dT}{T} - \frac{R}{P}dP$$

$$s_2 - s_1 = \int_{T_1}^{T_2} C_{p_0}\frac{dT}{T} - R\ln\left(\frac{P_2}{P_1}\right)$$

C_{v_0}, C_{p_0} are constants or equations from Appendix

- Net Entropy Change of the Universe

$$dS_{sys} \geq \frac{\delta Q}{T}$$

$$[S_2 - S_1]_{sys_{F\text{-}M}} \geq \int_{①}^{②} \frac{\delta Q}{T}$$

or

$$dS_{sys} = \frac{\delta Q}{T} + \delta S_{gn}$$

$$[S_2 - S_1]_{sys_{F\text{-}M}} = \int_{①}^{②} \frac{\delta Q}{T} + {}_1S_{gn\,2}$$

For the Universe (Net Entropy Change)

$$dS_{\underset{net}{universe}} = dS_{sys} + dS_{surroundings}$$

$$[S_2 - S_1]_{\underset{net}{universe}} = [S_2 - S_1]_{sys} + [S_2 - S_1]_{surrounding}$$

$$dS_{sur} = -\frac{\delta Q}{T_o} \quad ; \quad T_o = \text{Temp. of Surroundings}$$

$$[S_2 - S_1]_{sur.} = -\frac{{}_1Q_2}{T_o}$$

Principle of Net Entropy

$$dS_{\underset{universe}{net}} \geq 0, \qquad \begin{matrix} > \text{irreversible} \\ = \text{reversible} \end{matrix}$$

$$[S_2 - S_1]_{\underset{universe}{net}} \geq 0$$

- Second Law of Thermodynamics for Control Volumes.

$$\frac{dS_{cv}}{dt} + \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i \geq \sum \frac{\dot{Q}_{cv}}{T}$$

$>$ irreversible
$=$ reversible

- Second Law of Thermodynamics as applied to Surroundings

$$\frac{dS_{surr}}{dt} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e = \frac{-\dot{Q}_{cv}}{T_0}$$

- SSSF (Steady State, Steady Flow)

$$\frac{dS_{cv}}{dt} = 0$$

$$\frac{dE_{cv}}{dt} = 0$$

- USUF (Uniform State, Uniform Flow)

$\frac{dS_{cv}}{dt} \neq 0 \quad \therefore$ (therefor)

$\int_{t_i}^{t_e} dt$ integrate the entire equation

$$[[\underbrace{S_2 - S_1}_{\Delta S}] = \underbrace{(m_2 s_2 - m_1 s_1)_{cv}}_{\Delta S}] + \sum_e m_e s_e - \sum_i m_i s_i$$
$$\geq \int_{t_1}^{t_2} \sum_{cs} \frac{\dot{Q}_{cv}}{T} dt$$

$$(S_2 - S_1) + \sum_i m_i s_i - \sum_e m_e s_e = \frac{{}_1Q_{cv\,2}}{T_0}$$

- Rankine Cycle

> "The Rankine Cycle is a cycle that converts heat into work. The heat is supplied externally to a closed loop, which usually uses water. This cycle generates about 90% of all electric power used throughout the world, including virtually all solar thermal, biomass, coal, and nuclear power plants... The Rankine cycle is the fundamental thermo. underpinning of the steam engine. The Rankine cycle is sometimes referred to as a practical Carnot cycle because, when an efficient turbine is used, the T-S diagram begins to resemble the Carnot cycle."
>
> – "Rankine Cycle," Wikipedia

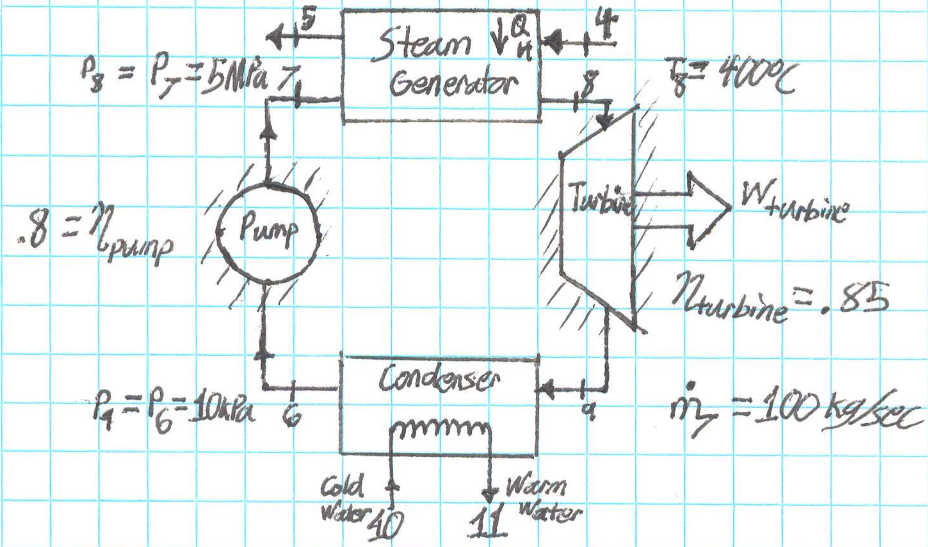
- Cycle:

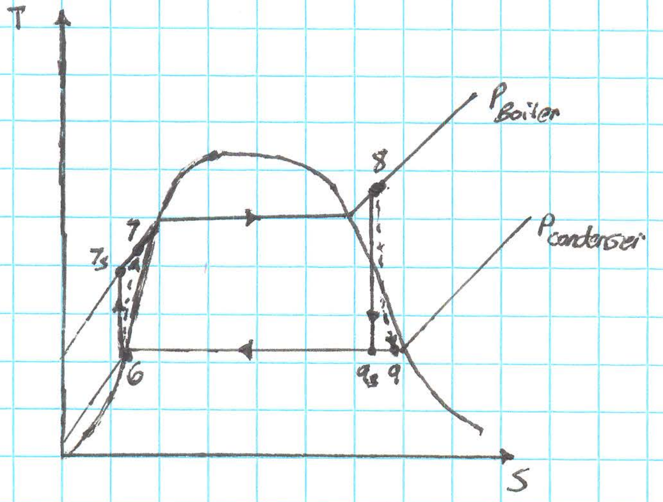
1. Isobaric Process; Q_H transfered to WF (Working Fluid) at P_H (Boiler/Heat Exchanger)
2. Adiabatic Process; WF expanded, $P_H \rightarrow P_L$ and Work is extracted (Turbine)
3. Isobaric Process; Q_L transfered from WF at P_L (Condenser)
4. Adiabatic Process; Work added to WF to raise $P_L \rightarrow P_H$ (Pump/Compressor)

- Ideal Rankine Cycle:

> "In an ideal Rankine cycle, the pump and turbine would be isentropic; i.e. the pump and turbine would generate no entropy and hence maximise the net work output."
>
> – "Rankine Cycle," Wikipedia

ex. Rankine Cycle





$s_{fg} = 7.501$

$s_f = .6492$ $\quad s_g = 8.15$

$h_f = 191.81$ $\quad h_g = 2584.63$

$h_{fg} = 2392.82$

$h_8 = 3195.64\ kJ/kg$

$s_8 = 6.6458\ kJ/kg\text{-}°K = s_{9_s}$

$$\eta_{turbine} = \frac{W_{turb}}{W_{turb\ isentropic}} = \frac{h_8 - h_9}{h_8 - h_{9_s}}$$

$$s_{9_s} = s_f + x_{9_s} \cdot s_{fg}$$

$$6.6458 = .6492 + x_{9_s} \cdot 7.501$$

$$x_{9_s} = .799$$

$h_{a_s} = 191.81 + (.799)2392.82 = 2104.73$

$$\eta_{turb.} = .85 = \frac{3195.64 - h_a}{3195.64 - 2104.73}$$

$h_a = 2268.37$

$x_a = .803$

$W_{turbine} = 927.27$ kJ/kg

$h_6 = 191.81$

$s_6 = .6492 = s_{7s}$

~~$s_{5MPa}(@40°C) = .5705$~~

~~$s_{5MPa}(@60°C) = .8284$~~

~~$\frac{.8284 - .5705}{20} = .012895$~~

~~$.6492 - .5705 = .0787/.012895 = 6.103$~~

~~$h_{5MP}(@40°C) = 171.95$~~

~~$h_{5MP}(@60°C) = 225.25$~~

~~$\frac{225.25 - 171.95}{20} = 2.665$~~

~~$2.665 \cdot 6.103 + 171.95 = 188.21 = h_{7s}$~~

$$\eta_{pump} = \frac{W_{pump\,rev}}{W_{pump\,irrev}} = \frac{h_6 - h_{7s}}{W_{pump\,irrev}}$$

$$W_{pump_{rev}} = -\int_{P_6}^{P_7} v(P)dP = -v_6\,(P_7 - P_6)$$

$v_6 = .00101$

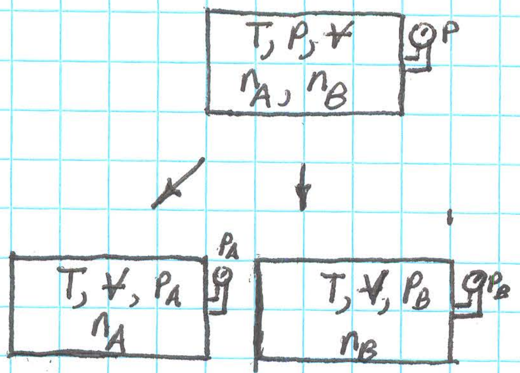
$$W_{pump_{rev}} = -5.0399\ kJ/kg$$

$$\eta_{pump} = .8 = \frac{-5.04}{W_{pump_{irrev}}}$$

$$W_{pump_{irrev}} = -6.3\ kJ/kg$$

Thermodynamics II

- Mixtures (Ideal Gas Mixtures)
  - Unmixing a Mixture



n_A = kmols of chemical A

$$P\forall = n\bar{R}T$$

$\bar{R}$ = Universal Gas Constant

$$P = \underbrace{P_A + P_B}_{\text{Partial Pressures}}$$ (Dalton's Law of Partial Pressures)

$y_A = \frac{n_A}{n}$ $\quad$ $y_B = \frac{n_B}{n}$ $\quad$ mole fractions

$$y_A + y_B = 1$$

$y_A \cdot P = P_A$ $\quad$ $y_B \cdot P = P_B$

⌜Note: This separation method does not work well with liquids.⌟

  - Properties of Mixtures

$$m = m_A + m_B + m_C + \ldots$$

$$n = n_A + n_B + n_C + \ldots$$

M_{mix} (Molecular Mass of Mixture) $= m/n$

$$R_{mix} = \bar{R} / M_{mix}$$

- Mixture of Dry Air (a) and Water Vapor (v)

$$\boxed{\begin{matrix} P, T, \forall \\ n_a, n_v \end{matrix}} \qquad P = P_a + P_v \qquad 1 = y_v + y_a$$

P_v = Partial Pressure of water vapor

Study of such a mixture is defined as Psychrometrics

$$y_v = \frac{P_v}{P}, \quad y_a = \frac{P_a}{P}$$

$$P_a \forall = m_a R_a T, \quad P_v \forall = m_v R_v T$$

ω (specific humidity, humidity ratio)

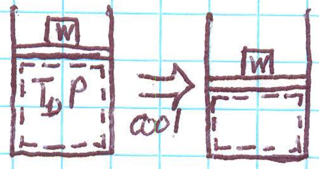
$$\omega = \frac{m_v}{m_a} = \frac{\text{mass (water) vapor}}{\text{mass dry air}} = \frac{n_v M_v}{n_a M_a}$$

$$= .622 \cdot \frac{n_v}{(n - n_v)} = .622 \frac{y_v}{(1 - y_v)}$$

$$y_v = \frac{P_v}{P}$$

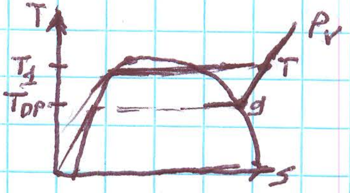
$$= .622 \frac{P_v}{1 - P_v}$$

T_{DP} (Dew Point Temperature) - Temperature at which vapor condenses



$W = P \cdot A$

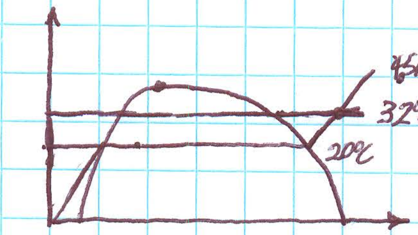
$T_{sat} = T_{DP}$
$P_v = y_v \cdot P$
Point at which first water dropplets form



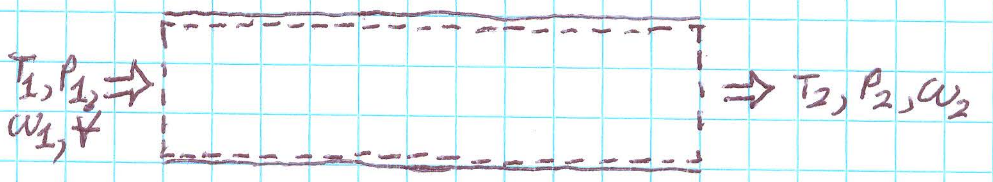
ϕ (Relative Humidity) $= \dfrac{P_v}{P_g(T)} = \dfrac{\text{Amount of Water Vapor held by Air}}{\text{Maximum possible water vapor}}$

Note: $P_g(T)$ is looked up in the steam tables and P_v generally given.

ex. $T_v = 20°C$, $P_{atm} = 100 kPa$
$P_v = 4.5 kPa$



- Flow Situations Involving Air and Water Vapor



$$\omega_1 = \frac{m_v}{m_a} = \frac{\dot{m}_v}{\dot{m}_a} = .622\,\frac{P_v}{P-P_v}$$

$$P\forall = mRT$$

$$\Downarrow$$

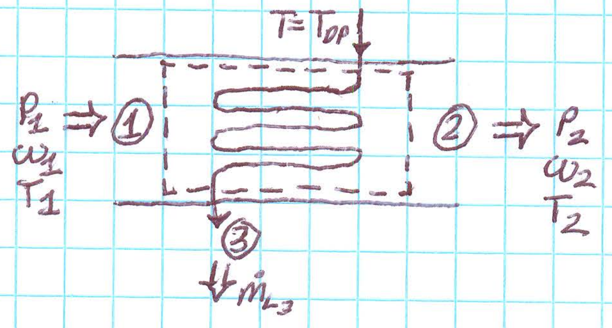
$$P\dot{\forall} = \dot{m}RT$$

$$\dot{m} = \frac{P\dot{\forall}}{RT}$$

$$\dot{m}_v = \frac{P_v\,\dot{\forall}}{R_v\,T} \qquad \dot{m}_a = \frac{P_a\,\dot{\forall}}{R_a\,T}$$

$$P = P_a + P_v \qquad P_v = y_v \cdot P$$

- ex. Mass Conservation (First Law C∀):



$$\omega_1 = \frac{\dot{m}_{v_1}}{\dot{m}_{a_1}} \Rightarrow \dot{m}_{v_1} = \omega_1 \cdot \dot{m}_{a_1}$$

- Dry Air:

$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

- Water Vapor:

$$\dot{m}_{v_1} = \dot{m}_{v_2} + \dot{m}_{L_3}$$

$$\omega_1 \cdot \dot{m}_a = \omega_2 \dot{m}_a + \dot{m}_{L_3}$$

$$\dot{m}_{L_3} = (\omega_1 - \omega_2)\dot{m}_a$$

condensation rate

- First Law $C\forall$ (SSSF):

$$\dot{Q}_{C\forall} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e + \cancelto{0}{\dot{W}_{C\forall}}$$

$$\dot{Q}_{C\forall} + \dot{m}_{v_1} h_{v_1} + \dot{m}_{a_1} h_{a_1} = \dot{m}_{v_2} h_{v_2} + \dot{m}_a h_{a_2} + \dot{m}_{L_3} \cdot h_{L_3}$$

$$\dot{Q}_{C\forall} + (\omega_1 h_{v_1} + h_{a_1}) \cdot \dot{m}_a = [\omega_2 h_{v_2} + \cancel{\dot{m}_a} h_{a_2} + (\omega_1 - \omega_2) \cdot h_{L_3}] \cdot \dot{m}_a$$

$$\dot{Q}_{C\forall} + (\omega_1 h_{v_1} + h_{a_1}) \cdot \dot{m}_a = [\omega_1 h_{L_3} + \omega_2 (h_{v_2} - h_{L_3}) + h_{a_2}] \cdot \dot{m}_a$$

$$(h_{a_2} - h_{a_1}) = (C_{p0})_a (T_2 - T_1)$$

Approximations for finding h_v, h_L

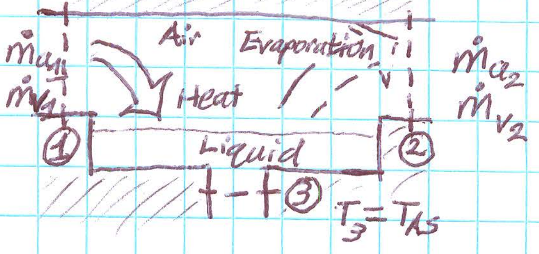
$h_{v_2} \cong h_g(T_2)$ Since Ideal Gas Behavior

$h_{v_1} \cong h_g(T_1)$

$h_{L_3} \approx h_f(T_3)$ Quick approx. for now

- Example

Given: $P_1 = 100\,kPa$, $T_1 = 30°C$, $T_{AS} = T_2 = T_3 = 20°C$



$$\dot m_{a_1} = \dot m_{a_2} = \dot m_a$$

$$\dot m_{v_1} + \dot m_{L_3} = \dot m_{v_2}$$

$$\omega_1 = \frac{\dot m_{v_1}}{\dot m_a}$$

$$\omega_2 = \frac{\dot m_{v_2}}{\dot m_a}$$

$$\dot m_a(\omega_1 - \omega_2) = -\dot m_{L_3}$$

$$\cancel{\dot Q_{cv}} \xrightarrow{} 0 + \left[\dot m_a h_{a_1} + \dot m_{v_1} h_{v_1} + \dot m_{L_3} h_{L_3}\right] = \dot m_a h_{a_2} + \dot m_{v_2} h_{v_2}$$

adiabatic

$$\cancel{\dot m_a} h_{a_1} + \omega_1 \cdot h_{v_1} \cdot \cancel{\dot m_a} + \cancel{\dot m_a}(\omega_2 - \omega_1) h_{L_3} = \cancel{\dot m_a}\omega_2 h_{v_2} + \cancel{\dot m_a} h_{a_2}$$

$$h_{a_1} + \omega_1 \cdot h_{v_1} + (\omega_2 - \omega_1) h_{L_3} = \omega_2 h_{v_2} + h_{a_2}$$

$$\omega_2 = .622 \frac{P_{v_2}}{P_2 - P_{v_2}} \qquad P_2 = P_1$$

$$\phi = 1 = \frac{P_{v_2}}{P_g(T_{AS})} \Rightarrow P_{v_2} = P_g(T_{AS}) = 2.339\,kPa$$

$$\omega_2 = .0149 > \omega_1$$

$$h_v = h_g(T_{AS}) = 2538\,kJ/kg$$

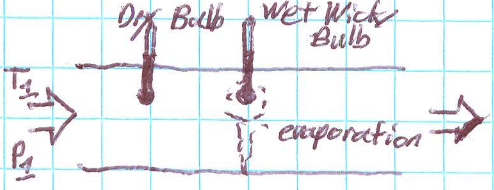
$$h_{a_1} - h_{a_2} = c_{p_0}(T_1 - T_{AS})$$

$$h_{L_3} = h_f(T_{AS}) = 83.94\,kJ/kg$$

$$\omega_1 = .010726$$

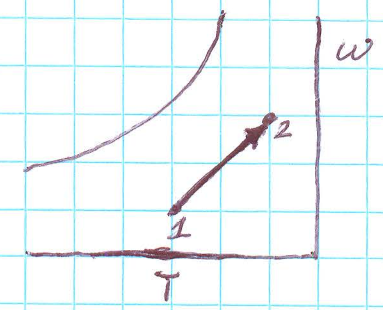
- Measurement of Humidity (ω)

Psychrometer:

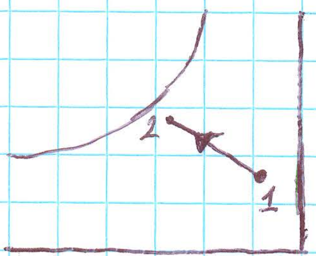


$$T_{AS} = T_{Wet Bulb} \approx T_{Air} + .2°C$$

Heating and Humidification:



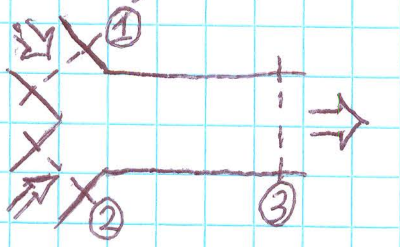
Adiabatic Humidifier (Desert Cooler/AC):



$\dot{Q} = 0$

Follows Wet Bulb Temp. Line

- Mixing of Air



$$\dot{m}_{a_1}, T_1, \omega_1, \dot{m}_{a_2}, T_2, \omega_2$$

$$\frac{\dot{m}_{v_1}}{\dot{m}_{a_1}} = \omega_1$$

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$$

$$\dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3}$$

$$\omega_1 \cdot \dot{m}_{a_1} + \omega_2 \cdot \dot{m}_{a_2} = \omega_3 (\dot{m}_{a_1} + \dot{m}_{a_2})$$

$$\cancelto{0}{\dot{Q}_{cv}} + \dot{m}_{a_1} h_1 + \dot{m}_{a_2} \cdot h_2 = (\dot{m}_{a_1} + \dot{m}_{a_2}) h_3 + \cancelto{0}{\dot{W}_{cv}}$$

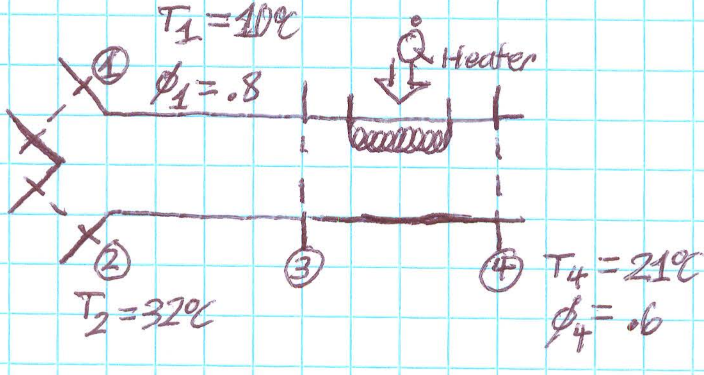
$$h_1 = h_{a_1} + \omega_1 h_{v_1}$$

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{h_3 - h_2}{h_1 - h_3} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3}$$

$$h_3 = \frac{\dot{m}_{a_1} \cdot h_1 + \dot{m}_{a_2} \cdot h_2}{(\dot{m}_{a_1} + \dot{m}_{a_2})}$$

$$\omega_3 = \frac{\dot{m}_{a_1} \cdot \omega_1 + \dot{m}_{a_2} \cdot \omega_2}{(\dot{m}_{a_1} + \dot{m}_{a_2})}$$

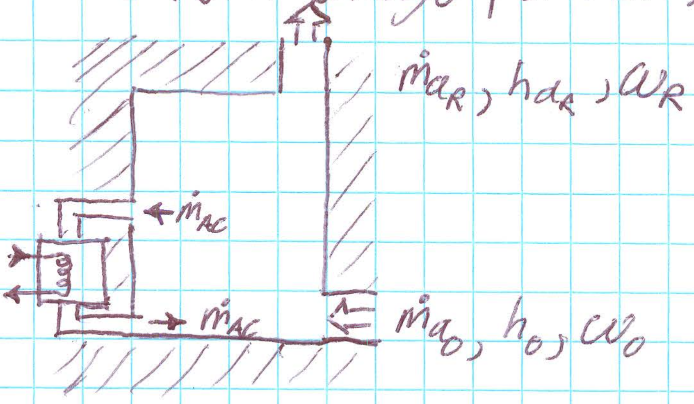
ex 1.108



$$\dot{Q}_{cv} + \dot{m}_{a1}\cdot h_1 + \dot{m}_{a2}\cdot h_2 = \dot{m}_{a3}\cdot h_3$$

- Infiltration Problem

ACH (Air Change per Hour) = $\dot{V}$



$$\cancelto{0}{\dot{Q}_{cv}} + \dot{m}_{a_0}(h_{a_0}) = \dot{m}_{a_R}(h_R)$$

$$(h)\dot{m}_{a_0} = \dot{m}_{a_R}(h) + \frac{dE_{cv}}{dt}$$

$$\frac{dE_{cv}}{dt} = \dot{m}_{a_R}\cdot h_R - \dot{m}_{a_0}\cdot h_{a_0}$$

[redacted] $\dot{m}_{a_R} = \dot{m}_{a_0} = \dot{m}_a$

$$\dot{Q}_{cv} = \dot{m}_a(h_R - h_{a_0})$$

ex. $T_{Room} = 25°C$

$\phi_R = .5 =$ [redacted] ϕ_0

$T_0 = 35°C$

$\dot{m}_a = .05\ kg/s$

$\dot{Q}_{cv} = \dot{m}_a(h_R - h_0)$

$= .05(70 - 100)$

$P\dot{V} = \dot{m}RT$

- Thermodynamics Relationships [(Non)Ideal Substances]
  - Genesis of Thermodynamics Tables

$$\begin{tikzcd}
\text{Properties} \arrow[d] \arrow[dr] & \\
\text{Measurable} & \text{Inmeasurable}
\end{tikzcd}$$

$$P, \forall, T, C_p, C_v \underset{\text{Thermo Relations}}{\Rightarrow} S, U, H$$ [Can't measure chaes]

For Pure Substances

$$\forall = \forall(T, P)$$

$$s = s(T, P)$$

$$\left.\frac{\partial z}{\partial x}\right)_y \left.\frac{\partial x}{\partial y}\right)_z \left.\frac{\partial y}{\partial z}\right)_x = -1$$

$$\left.\frac{\partial z}{\partial x}\right)_y = \left.\frac{\partial z}{\partial \omega}\right)_y \left.\frac{\partial \omega}{\partial x}\right)_y$$

$$\left.\frac{\partial s}{\partial v}\right)_p = \frac{C_p}{T} \cdot \left.\frac{\partial T}{\partial v}\right)_p$$

α_p = Coefficient of Vol. Expansivity

$$= \frac{1}{v} \frac{\partial v}{\partial T}$$

$\therefore$

$$\left.\frac{\partial s}{\partial v}\right)_p = \frac{C_p}{T \cdot v \cdot \alpha_p}$$

- Maxwell's Relation Equations

$$\left[\begin{array}{l} Tds = du + Pdv \\ Tds = dh - vdP \end{array}\right.$$

$g \equiv h - T \cdot s$ (Gibb's Function)

$a \equiv u - T \cdot s$ (Helmholtz Function)

$dg = dh - Tds - sdT$

$dg = vdP - sdT$

$da = du - Tds - sdT$

$da = -Pdv - sdT$

$\rightarrow du = Tds - Pdv$

$\rightarrow dh = Tds + vdP$

$dg = vdP - sdT$

$da = -Pdv - sdT$

1. $\left.\frac{\partial T}{\partial v}\right)_s = -\left.\frac{\partial P}{\partial s}\right)_v$

2. $\left.\frac{\partial T}{\partial P}\right)_s = \left.\frac{\partial v}{\partial s}\right)_P$

3. $\left.\frac{\partial s}{\partial v}\right)_T = \left.\frac{\partial P}{\partial T}\right)_v$

4. $-\left.\frac{\partial s}{\partial P}\right)_T = \left.\frac{\partial v}{\partial T}\right)_P = \alpha_P \cdot v$

$$u = u(T,v)\,;\; s = s(T,v)$$

$$du = T\,ds - P\,dv$$

$$du = \left.\frac{\partial u}{\partial T}\right)_v dT + \left.\frac{\partial u}{\partial v}\right)_T dv$$

$$ds = \left.\frac{\partial s}{\partial T}\right)_v dT + \left.\frac{\partial s}{\partial v}\right)_T dv$$

$$du = du$$

$$\left.\frac{\partial u}{\partial T}\right)_v dT + \left.\frac{\partial u}{\partial v}\right)_T dv = T\left[\left.\frac{\partial s}{\partial T}\right)_v dT + \left.\frac{\partial s}{\partial v}\right)_T dv\right] - P\,dv$$

$$T\left.\frac{\partial s}{\partial v}\right)_T - \left.\frac{\partial u}{\partial v}\right)_T - P = 0$$

$$\left.\frac{\partial u}{\partial T}\right)_v - T\left.\frac{\partial s}{\partial T}\right)_v = 0$$

$$\Downarrow$$

$$c_v = T\left.\frac{\partial s}{\partial T}\right)_v$$

$$\left.\frac{\partial u}{\partial v}\right)_T = T\left.\frac{\partial s}{\partial v}\right)_T - P$$

$$= T\left.\frac{\partial P}{\partial T}\right)_v - P$$

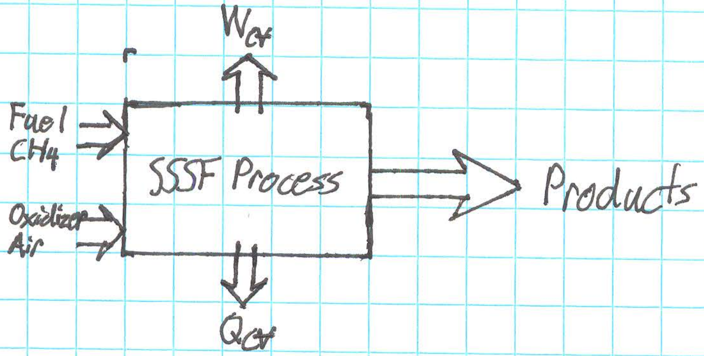
$$\left.\frac{\partial h}{\partial P}\right)_T = v - T\frac{\partial v}{\partial T} \quad ; \quad c_P = \left.\frac{\partial h}{\partial T}\right)_P$$

$$= \left.\frac{\partial s}{\partial T}\right)_P$$

- ■ First Law Analysis of Chemical Reacting

$$Q_{cv} + H_{Reactant} = H_{Product} + W_{cv}$$

kJ/(kmol of molecules)



$$Q_{cv} + \sum_i n_i(\bar{h}_f^o + \Delta\bar{h})_i = \sum_j n_j(\bar{h}_f^o + \Delta\bar{h})_j + W_{cv}$$

- Equivalence Ratio (ϕ)

$$\phi = \frac{(FA)_{Act}}{(FA)_{Stoichiometric}} = \frac{(AF)_{stoich}}{(AF)_{Act}}$$

- Approximation of Air (Atmospheric)

Air $\approx$ 21% O_2

79% N_2

As O_2 is the Oxidizing molecule out of air, normally reaction equations are based via every mole of O_2. There is 1 mole O_2 in every 4.76 moles of air.

ex.

$$CH_4 + 2x(O_2 + 3.76N_2) \Rightarrow CO_2 + 2H_2O +$$

$$2(-1+x)O_2 + 2x(3.76N_2)$$

x = 1 for theoretical air

- $\underset{FA}{\text{Fuel-Air}}$ vs $\underset{AF}{\text{Air-Fuel}}$ Ratio

$$FA_{mole} = \frac{n_{fuel}}{n_{air}} = \frac{1}{AF_{mole}}$$

$$FA_{mass} = \frac{m_{fuel}}{m_{air}} = \frac{1}{AF_{mass}}$$

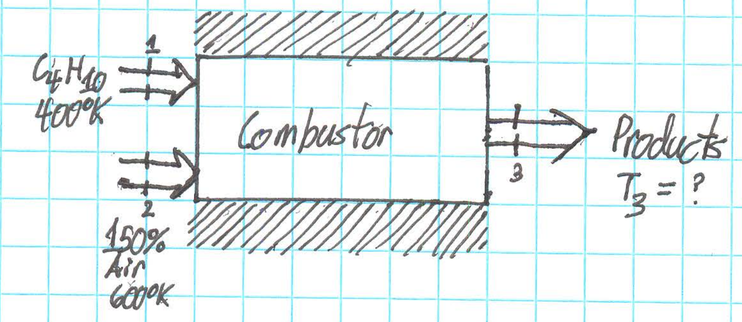
$$AF_{mole} = \frac{n_{air}}{n_{fuel}}$$

$$* AF_{mass} = \frac{m_{air}}{m_{fuel}}$$

- Note: For [redacted] ideal gas mixtures, mole fractions are equal to volume fractions

ex. Adiabatic Flame Temp.

Butane (C_4H_{10}) gas at 400°K is mixed with 150% theoretical air at 600°K and is burned in an adiabatic SSSF combustor



$$C_4H_{10} + 6.5x(O_2 + 3.76N_2) \Rightarrow 4\,CO_2 + 5\,H_2O +$$

$$6.5(-1+x)\,O_2 + 6.5x(3.76N_2)$$

$x = 1.5$ (Theoretical Air)

$Q_{CV} = W_{CV} = 0$ (since adiabatic and SSSF)

$$\cancelto{0}{Q_{cv}} + \underbrace{\sum_i n_i (\bar{h}_f^o + \Delta\bar{h})_i}_{H_R} = \underbrace{\sum_e n_e (\bar{h}_f^o + \Delta\bar{h})_e}_{H_P} + \cancelto{0}{W_{cv}}$$

$$H_R = n_{C_4H_{10}} (\bar{h}_f^o + \Delta\bar{h})_{C_4H_{10}} +$$

$$n_{O_2} (\bar{h}_f^o + \Delta\bar{h})_{O_2} + n_{N_2} (\bar{h}_f^o + \Delta\bar{h})_{N_2}$$

$\bar{h}^o_{f_{C_4H_{10}}} = -126200$ kJ/kmol (A-10)

$\bar{h}^o_{f_{O_2}} = \bar{h}^o_{f_{N_2}} = 0$

$\Delta\bar{h}_{O_2} = 9245$ kJ/kmol (600°K)

$\Delta\bar{h}_{N_2} = 8894$ kJ/kmol (600°K)

$\Delta\bar{h}_{C_4H_{10}} = M \cdot C_{p_0} \cdot \Delta T$

$= 58.124 \text{ kg/kmol} \cdot 1.716 \text{ kJ/kg-°K} \cdot (400 \rightarrow 298 \text{ °K})$

$= 10173.56$ kJ/kmol

$$H_R = 1 \cdot (-126200 + 10173.56) +$$

$$9.75(9245) + 36.66(8894)$$

$= 300166.35$ kJ

$$H_P = n_{CO_2} (\bar{h}_f^o + \Delta\bar{h})_{CO_2} + n_{H_2O} (\bar{h}_f^o + \Delta\bar{h})_{H_2O} +$$

$$n_{O_2} (\cancel{\bar{h}_f^o} + \Delta\bar{h})_{O_2} + n_{N_2} (\cancel{\bar{h}_f^o} + \Delta\bar{h})_{N_2}$$

$\bar{h}^o_{f_{CO_2}} = -393522$ kJ/kmol

$\bar{h}^o_{f_{H_2O}} = -241826$ kJ/kmol

$n_{CO_2} = 4$ $n_{H_2O} = 5$ $n_{O_2} = 3.25$ $n_{N_2} = 36.66$

$$H_p = -2783218 + 4 \cdot \Delta\bar{h}_{CO_2} + 5 \cdot \Delta\bar{h}_{H_2O} +$$

$$3.25 \cdot \Delta\bar{h}_{O_2} + 36.66 \cdot \Delta\bar{h}_{N_2}$$

est. $T_e = 2000°K$

$H_p = 196782.42$

est. $T_e = 2200°K$

$H_p = 586648.42$

$\Delta H_p (2000-2200°K) = 389866$

$300166.35 - 196782.42 = 103383.58$

$T_3 \simeq 2053.035°K$

$$y_{H_2O} = \frac{n_{H_2O}}{n_{Total}} = \frac{5}{48.91} = .1022$$

$P_{H_2O} = y_{H_2O} \cdot P$

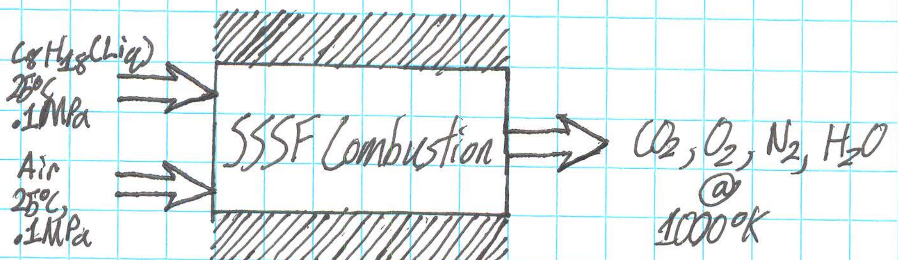
$= .1022 \cdot 100 kPa$

$= 10.22 kPa$

$T_{Dew Point} \simeq 45.81°C$

ex. Fuel/Air Ratio Calculation

In a combustor using octane (C_8H_{18}), it is desired to limit the exhaust temp. to 1000°K by using air in excess of the stoichiometric amount.



$$C_8H_{18} + 12.5x(O_2 + 3.76N_2) \Rightarrow 8CO_2 + 12.5(x-1)O_2 + 9H_2O + 12.5x(3.76N_2)$$

$$\cancelto{0}{Q_{cv}} + H_R = H_P + \cancelto{0}{W_{cv}}$$

$$\therefore \quad H_R = H_P$$

$$H_R = 1(\bar{h}_f^o + \cancel{\Delta\bar{h}})_{C_8H_{18}} + \cancelto{0}{12.5x(\bar{h}_f^o + \Delta\bar{h})_{O_2}} + \cancelto{0}{12.5\cdot 3.76\cdot x(\bar{h}_f^o + \Delta\bar{h})_{N_2}}$$

$= \sout{-208600 \text{ kJ/kmol}}$ -250105 kJ/kmol

note: Liq. state

$$H_P = 8(\bar{h}_f^o + \Delta\bar{h})_{CO_2} + 12.5(x-1)(\bar{h}_f^o + \Delta\bar{h})_{O_2} + 9(\bar{h}_f^o + \Delta\bar{h})_{H_2O} + 12.5\cdot x\cdot 3.76(\bar{h}_f^o + \Delta\bar{h})_{N_2}$$

$$= 8(-393522 + 33397) + 12.5(x-1)(22703) + 9(-241826 + 26000) + 47\cdot x(21463)$$

$= -5034034 + (12.5x - 12.5)(22703)$

$+ 47x(21463)$

$= -5317821.5 + 283787.5x$

$+ 1008761x$

$5067716.5 = 1292548.5x$

$x = 3.92$

$$FA_{mass} = \frac{114.232}{18.66 \cdot 28.97 \cdot 12.5}$$

$$= \frac{114.232}{6756.96} = .0169$$

$$AF_{mass} = \frac{1}{FA_{mass}} = 59.151$$

High Performance Piston Engines (MAE 477)

- Bore - diameter of cylinder in a piston engine
- Stroke - action of the piston traveling the length of the cylinder, is also the distance from Top Center (TC) to Bottom Center (BC)
- Clearance Volume ($\forall_c$) - minimum cylinder volume when the piston is at Top Center (TC) position
- Displacement (Volume) ($\forall_d$) - volume displaced by a single sweap of the piston from Top Center to Bottom Center (BC).
    - Engine Displacement - volume displaced by all cylinders and pistons in an engine.
- Compression Ratio (r_c) - $\dfrac{(\forall_c + \forall_d)}{\forall_c}$ Ratio of maximum volume to minimum volume
- Mean Piston Speed - average speed in a reciprocating engine.
  $MPS = 2 \cdot \text{Stroke (Length)} \cdot RPM/60$ [For 4-Stroke Engine]
- Brake (Horse) Power - measure of an engine's horse power before loss in power caused by gearbox, alternator, differential, water pump, and/or other auxiliary components.
  $$P(HP) = \frac{N(\text{rev/min}) \cdot T(\text{lbf-ft})}{5252}$$
  $$P(kW) = 2\pi \cdot N(\text{rev/s}) \cdot T(\text{N-m}) \cdot 10^{-3}$$
    - Dynamometer - device for measuring force, moment-force (τ - torque), and/or power
- Relationship between torque and power -
  $$P(W) = \tau(\text{N-m}) \cdot 2\pi \cdot N(\text{rev/s})$$
  $$P(HP) = \frac{\tau(\text{lbf-ft}) \cdot 2\pi \cdot N(\text{rpm})}{33000}$$
- Brake Thermal Efficiency - Thermal Efficiency, the ratio of brake power output to power output.

- Brake Mean Effective Pressure - Mean effective pressure calculated from brake power.

  P_{BMEP} (Metric) $= \frac{\tau \cdot n_c}{\forall_d} \cdot 2\pi$; n_c = number of revolutions per cycle

  = 2 for a 4 stroke engine

  P_{BMEP} (Imp.) $= \frac{\tau(\text{lbf-ft}) \cdot n_c \cdot 75.4}{\forall_d\ (\text{in}^3)}$

- Volumetric Efficiency ($n_\forall$) - volume flow rate of air into intake / rate volume displaced by piston.

- WOT (Wide Open Throttle) - refers to an engine's maximum intake of air and fuel.

- Naturally Aspirated - No force (air) induction, not super/turbo charged

- Supercharged - forced (air) induction via mechanical power off the crank shaft. Provides instant boost when motor requires, but can siphon up to 1/3 power off crankshaft.

- Turbocharged - specific type of supercharged system, which utilizes exaust gas to run the forced induction compressor. This provides a much more efficient boost, but has a lag between the exaust and the boost

- Brake specific fuel consumption (BSFC) - r/P

  r = rate of fuel consumption (g/s)

  P = power produced in watts = τ (N-m) $\cdot\ \omega$ (rad/s

[1 Gallon (US) = $3.785 \cdot 10^{-3}\ m^3$

[120 kmph = 74.5 mph

- Ideal Otto Cycle

  Rev. Adiabatic Compression (of air)

  Instantaneous Combustion (constant volume)

  Rev. Adiabatic Expansion (power stroke)

  Instantaneous Heat Rejection (exaust)

ex. Injector for Fuel (Coyote Engine)

60μm diameter droplets

$$\forall_{drop} = \frac{\pi d^3}{6}$$

$$m_{drop} = \rho \forall_d$$

106,000 (60μm drops/stroke)

$$\frac{P}{\rho} = RT \qquad \rho_{gas(oline)} \simeq 4.47 kg/m^3$$

$$\rho_{air} = \underline{1}.17 kg/m^3$$

$$P_{gas} = \gamma_{gas} \cdot P$$

55.46 moles of Air per mole of gas

56.46 total moles

$$\gamma_{gas} = \frac{1}{56.46} = 1.77\%$$

$$\frac{3000 rpm}{2 \text{ strokes/cycle}} \cdot (4.951 \cdot 10^{-3}) m^3 = 7.4265 \ m^3/min \cdot 1.17 \ kg/m^3$$

ex. Coyote Engine Cont.

CR (r_c) = $\underline{11}:\underline{1}$

Intake Air: $T_{air} = 77°F\ (25°C) = \underline{298.15}°K$

$P_1 = \underline{100 kPa}$

Estimation of Temp. at Top Center

$P\forall^{\gamma} = C$ for ideal gas

$$P_1 \forall_1^{\gamma} = P_{TC} \forall_{TC}^{\gamma}$$

$$P_{TC} = P_1 \left(\frac{\forall_1}{\forall_{TC}} \right)^{\gamma}$$

$$\frac{\forall_1}{\forall_{TC}} = \frac{(\forall_d + \forall_c)}{\forall_c} = CR = \underline{11}$$

$$\gamma_{air} = 1.4$$

$$P_{TC} = 100\ (11)^{1.4}$$

$$= 2.87\ MPa\ (4.16 psi)$$

$$T_1 \forall_1^{\gamma-1} = T_{TC} \forall_{TC}^{\gamma-1}$$

$$T_2 = 778°K$$

4.<u>11</u> gasoline engine

isooctane / air ($C_8 H_{18}$)

$T_{in} = 25°C = 298.15°K$ $W_{out} = 50\,kW$

$\dot{m}_{fuel} = 3 g/s$ $T_{out} = 660°K$

Vol. or Mole frac.

CO_2 : 11.4%

O_2 : 1.6%

CO : 2.9%

N_2 : 84.1%

$$\alpha C_8 H_{18} + \beta x (O_2 + 3.773 N_2) \Longrightarrow$$

$$\zeta (x-1) O_2 + \varepsilon CO_2 + \delta CO +$$

$$\gamma x (3.773 N_2) + [\omega H_2O]$$

C: $8 \cdot \alpha = \varepsilon + \delta$

$= .114 + .029$

$\alpha = .01788$

H: $18 \alpha = 2 \omega$

$\omega = .161$

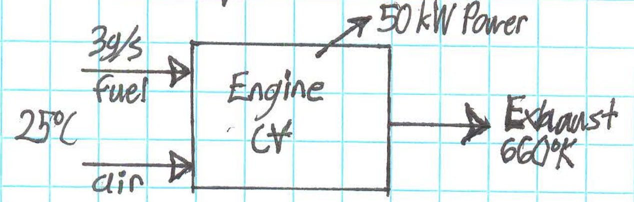
N: $\beta = \gamma$ $\beta \cdot x \cdot 3.773 = .841$

$\beta x = .223$

O: $\zeta (x-1) = .016$

$.223 \cdot 2 = .016 \cdot 2 + .114 \cdot 2 + .029 + .161$

✓



Enthalpy of air @ 25°C = 0

$\bar{h}_{C_8H_{18}} = -259280 \text{ kJ/kmol}$

$M_{C_8H_{18}} = 12.01 \cdot 8 + 18 = 114.08 \text{ kg/kmol}$

$$\frac{\dot{m}_{fuel}}{M_{fuel}} = \frac{.003 \text{ kg/s}}{114.08 \text{ kg/kmol}} = .0000263 \text{ kmol/s}$$

$$= .0263 \text{ mol/s}$$

- Saturated Systems:

$x = m_{vap}/m$ (Quality)

$v = (1-x)v_f + x \cdot v_g$ (Specific Volume)

$W = \int_1^2 F\,dx$ (work, J = N-m, ft-lb)

$\dot{W} = \text{Power} = dW/dt$ (Power, Watt ($W = J/s$), hp (ft lbf/s))

$\partial W = P \cdot A\,dL$ (Work for a piston) $= P_{ext}\,dV$

$\dot{Q} = A \cdot h \cdot \Delta T$

- Control Mass Systems (CM):

$U_2 - U_1 + \underbrace{\tfrac{1}{2} m (V_2^2 - V_1^2)}_{\Delta KE} + \underbrace{mg(z_2 - z_1)}_{\Delta PE} = {}_1Q_2 - {}_1W_2$

1st Law of Thermo

$E_2 - E_1 = {}_1Q_2 - {}_1W_2$

- Control Volume Systems (CV):

$q_{cv} + h_1 + \tfrac{1}{2} V_1^2 + g Z_1 = w_{cv} + h_2 + \tfrac{1}{2} V_e^2 + g Z_2$

$q = \dot{Q}_{cv}/\dot{m}$ $\quad w = \dot{W}/\dot{m}$ (SSSF only)

$\dot{W}_{flow} = P\dot{V} = \dot{m} P v$

$\dot{m} = \int \rho V\,dA = AV_v$

$\frac{dS_{cv}}{dt} + \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i \geq \sum_{cs} \frac{\dot{Q}_{cv}}{dt}$

- Second Law of Thermo:

Heat Engine: η - Thermal Efficiency

$W_{HE} = Q_H - Q_L$; $\eta_{HE} = W_{HE}/Q_H = 1 - \frac{Q_L}{Q_H}$

$\eta_{HE} = W_{HE}/Q_H \leq \eta_{carnot\ HE} = 1 - T_L/T_H$

Heat Pump: β - coefficients of Performance

$W_{HP} = Q_H - Q_L$; $\beta_{HP} = Q_H/W_{HP} = \frac{Q_H}{Q_H - Q_L}$

$\beta_{HP} = \frac{Q_H}{W_{HP}} \leq \beta_{carnot_{HP}} = T_H/T_H - T_L$

Refridgerator:

$W_{REF} = Q_H - Q_L$; $\beta_{REF} = Q_L/|W_{REF}| = \frac{Q_L}{|Q_H - Q_L|}$

$\beta_{REF} = \frac{Q_L}{W_{REF}} \leq \beta_{carnot_{REF}} = \frac{T_L}{T_H - T_L}$

Carnot Cycle: (Reversible)

1. Isothermal; $Q_H \rightarrow WF @ T_H$
2. Adiabatic; $W_F \rightarrow T_H \rightarrow T_L$
3. Isothermal; $Q_L \rightarrow WF @ T_L$
4. Adiabatic; $WF, T_L \rightarrow T_H$

WF - Working Fluid

Rankine Cycle: (Reversible)

1. Isobaric; $Q_H \rightarrow WF @ P_H$
2. Adiabatic; (s=const.) $P_H \rightarrow P_L$
3. Isobaric; $Q_L \leftarrow WF @ P_L$
4. Adiabatic; (s=const.) $P_L \rightarrow P_H$

Adiabatic:

$Q = 0 = q_c$

Single In/out: (SSSF)

$\dot{m}_i = \dot{m}_e = \dot{m}$

* Max Theoretical Value:

$s_e = s_i$

Ideal Gas:

Isentropic (s=const):

$Pv^k = \text{const.}$; $k = C_{P_0}/C_{V_0}$

Const. Specific Heat: ($k = n$)

$P_2/P_1 = (V_1/V_2)^n = (T_2/T_1)^{n/n-1}$

$n = 0$; P = const. (Isobaric)

$n = 1$; T = const. (Isothermal)

$n = k$; s = const. (Isentropic)

$n = \infty$; v = const. (Isochoric)

$u_2 - u_1 = \int_{T_1}^{T_2} C_{V_0}(T)\,dT$

$h_2 - h_1 = \int_{T_1}^{T_2} C_{P_0}(T)\,dT$

$h = u + RT$

$s_2 - s_1 = C_{V_0} \ln(T_2/T_1) + R \ln(v_2/v_1)$

$s_2 - s_1 = C_{P_0} \ln(T_2/T_1) - R \ln(P_2/P_1)$

SSSF, neglect ΔPE, ΔKE, single In/out:

No Work (condenser/generator):

${}_1q_2 = h_2 - h_1$

No Heat (turbine/pump):

${}_1W_2 = h_1 - h_2$

$W_{cv} = v(P_i - P_e)$ (Incompressible flow)
($\rho = 1/v$ = const, Reversible)

Net Entropy Change of the Universe: (42)

$dS_{\underset{net}{universe}} = dS_{sys} + dS_{surroundings}$

$[S_2 - S_1]_{\underset{net}{universe}} = [S_2 - S_1]_{sys} + [S_2 - S_1]_{sur}$

$[S_2 - S_1]_{sur} = -{}_1Q_2/T_0$

$dS_{\underset{net}{universe}} \geq 0$ $\quad$ > irreversible
= reversible

SSSF:

$\frac{dS_{cv}}{dt} = 0$

** Isentropic Efficiency of a Device (s=const.):

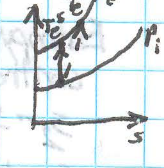
Isentropic Efficiency of a Compressor or Pump:

$\eta_{\binom{c}{p}} = W^s_{\binom{c}{p}} / W^a_{\binom{c}{p}}$ $\quad$ a=actual; s=isentropic

$W_{cv} = h_i - h_e$

$W^a_{\binom{c}{p}} = h_i - h_e^a$

$W^s_{\binom{c}{p}} = h_i - h_e^s$



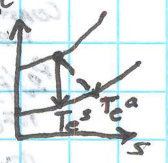
$= \frac{h_i - h_e^s}{h_i - h_e^a} \underset{\text{ideal gas only}}{=} \frac{(T_i - T_e^s)}{(T_i - T_e^a)}$

cont. on back ↓

Isentropic Efficiency of a Turbine:

$$\eta_t = \frac{W_t^a}{W_t^s} \underset{\substack{\text{SSSF, Single in/out} \\ \text{Adiabatic, Neglect} \\ \Delta PE, \Delta KE}}{=} \frac{h_i - h_e^a}{h_i - h_e^s}$$

$$\underset{\substack{\text{ideal} \\ \text{gas}}}{=} \frac{(T_i - T_e^a)}{(T_i - T_e^s)}$$



Isentropic Efficiency of a Nozzle:

$$\eta_N = \frac{(V_e^2)^a}{(V_e^2)^s} \underset{\substack{\text{SSSF, single in/out} \\ \text{Adiabatic, Neglect} \\ \Delta PE, \text{ NOT } \Delta KE}}{=} \frac{(h_i - h_e^a) + \left(\frac{V_i^2}{2000}\right)^a}{(h_i - h_e^s) + \left(\frac{V_i^2}{2000}\right)^s}$$

$$\underset{\substack{\text{ideal} \\ \text{gas}}}{=} \frac{(T_i - T_e^a)}{(T_i - T_e^s)}$$

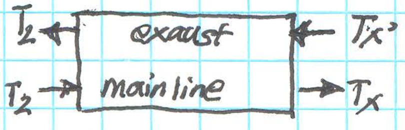
* Throttling Valve (Expansion Valve):

No Work/Heat
$h_i = h_e$; $h = \text{const}$
$s_e > s_i$

Brayton Cycle - Gas turbine
Similar to jet engine

Regenerator

$$\eta_{regen} = \frac{T_x - T_2}{T_{x'} - T_2}$$



Work:

Reversible Pump:

$$W_{pump\,rev} = -\int_{P_1}^{P_2} v(P)\,dP$$

$$= \int_{V_1}^{V_2} P(V)\,dV$$

Enthalpy (h) = u + Pv

$$C_v = \frac{\partial u}{\partial T} \qquad C_p = \frac{\partial h}{\partial T}$$

$$u_2 - u_1 = \int C_v\,dT \approx C_{v_0}(T_2 - T_1)$$

$$h_2 - h_1 = \int C_p\,dT \underset{\substack{\text{ideal} \\ \text{gas}}}{\approx} C_{p_0}(T_2 - T_1)$$

$h = u + RT$ (ideal gas)

Entropy:

Control Mass (du) Ideal Gas

$$s_2 - s_1 = \int_{T_1}^{T_2} C_{v_0} \frac{dT}{T} + R\ln\left(\frac{V_2}{V_1}\right)$$

Control Volume (dh) Ideal Gas

$$s_2 - s_1 = \int_{T_1}^{T_2} C_{p_0} \frac{dT}{T} - R\ln\left(\frac{P_2}{P_1}\right)$$

$1\,m^3 = 1000\,L$
1 BTU = 1 lb (H_2O) raised 1°F
1 Cal = 1 g (H_2O) raised 1°C
1 kCal = 4.18 kJ
1 kJ/hr = 3599.71 kW

$$\eta_{wind\,turbine} = \left[1 - \frac{V_2^2}{V_1^2}\right]$$

ENTROPY IS PATH INDEPENDANT